sequence for his propositions or some new foundation on which to build,—to such a teacher these notes will seem like the words of one having authority and not as those of the educational scribes and Pharisees.

Not least among the valuable features of the work is the index of Greek terms and the index of proper names, aids which readers so often miss in books of this general nature.

In America the book will serve an immediate purpose, in that it is one of the few books on geometry that no teacher can afford to be without, that is indispensable in the library of any well-equipped high school, and that the general reader with scholarly taste will welcome as a pleasant relief from most of our current educational literature. But it is also to be hoped that it will serve still another purpose, the one already referred to as supplying a new classic for those elective courses which may very likely come with the development of a better and more modern type of senior high school in this country.

DAVID EUGENE SMITH.

Theory of Maxima and Minima. By HARRIS HANCOCK. Boston, Ginn and Company, 1917. xiv + 194 pages. Octavo. Price $2.50.

The little treatise deals with maxima and minima, the first half being concerned principally with the case of functions of two variables. The first chapter disposes effectually of the one dimensional problem, and some sections treat in particular examples of three independent variables. The second half, chapters V to VIII, treat the general case. Ordinary and extraordinary maxima and minima, the latter occurring only at irregular points of the function are considered, and considerable space is given relative extremes in addition to the usual discussion of absolute extremes. The discussion treats the problem from many points of view and is particularly rich in elements common to other mathematical disciplines.

The principal topics emphasized are homogeneous forms, definite, semidefinite, and indefinite, and their relation to the determination of maxima and minima; the fallacy of the Lagrangian criterion by which the point in question is approached on straight lines only; various attempts to improve the theory particularly those of Stolz, Scheefer, and v. Dantscher; homogeneous quadratic forms in many variables; the treatment of conditions by obvious and also by more symmetric methods; applications to geometrical and physical problems; subsidiary conditions as inequalities rather than equations of condition; Gauss's principle in problems of mechanics; reversion of series; certain fundamental conceptions in the theory of analytic functions, such as analytic and algebraic dependence, and algebraic singularities.

The readers of the MONTHLY are doubtless most interested in the question as to the place in a college curriculum this book may be expected to fill. The author mentions in his preface, "As introductory to a courese of lecture on the calculus of variations, I have for a number of years given a brief outline of the

theory of maxima and minima." It must not be supposed that the book under discussion depends for its completeness on any subsequent course in the calculus of variations. It should be regarded rather as a self-contained exposition of an important elementary subject too seldom mentioned as an independent discipline but one which despite its present fairly obvious form, has been a subject of error on the part of many of the most conspicuous modern mathematicians, such as Lagrange, Bertrand, Serret, etc. The treatment is made concrete by numerous numerical examples of no little interest on their own account, and of the sort which usually appeal to undergraduate students. The reviewer regards the book as a very eligible candidate for a short post-calculus course. It does not require the acumen essential in an efficient course in real variables nor does it demand as much time as a fair course in elementary differential equations, while it shares with the former the chance of developing the critical faculties of the student and with the latter the numerical concreteness desired by him.

If the calculus of variations is to be taught, such an elementary view as given in Byerly's little course *An Introduction to the Calculus of Variations*, 1917, needs no book of the sort under review. On the other hand, Bolza's well-known *Lectures on the Calculus of Variations* presupposes a familiarity with analysis, and particularly with the notions of real variables for which the present book is in no sense a sufficient introduction. The author's remark "a treatment of these cases, the extraordinary cases, required more study than was anticipated," merely illustrates the fact that the treatment of the case of a finite number of variables involves difficulties of its own and is not to be viewed as a mere preface to the calculus of variations. That these difficulties are not unworthy of attention the book amply demonstrates.

The reviewer noted several features which he regards as unfortunate in a treatment of this sort, of which the most important are as follows. The student is required to have a precise familiarity with the notion of continuity, and the idea of uniform continuity is in essence involved many times but no word of caution is given to such students as may be somewhat hazy as to the exact definition. While the theorems are in general given neither name nor number, we note "Stolz's added theorem," "The Scheefer theorem otherwise stated" (which fills nearly a page), as unfortunate names for fundamental theorems. "A function $f(x)$ is *regular* in the neighborhood of the position $x = a$, if the function in this neighborhood has everywhere a definite value which changes in a continuous manner with x," p. 73, and here real variables only are under consideration! "Legendre was able indeed to show that this sum could not be greater than two right angles; however he did not show *they* could not be less than two right angles," p. 135, italics not in the text. "If a continuous variable quantity is defined in any manner this quantity has an *upper* and a *lower* limit; that is, there is a definitely determined quantity g of such a kind that no value of the variable can be greater than g, although there is a value of the variable

which can come as near to g as we wish . . .," p. 136, while the character of the region of definition is treated as wholly arbitrary.

ALBERT A. BENNETT.

Jahrbuch über die Fortschritte der Mathematik . . . herausgegeben. Von E. LAMPE† und A. KORN. Band 45. Jahrgang 1914-1915. (In 3 Heften) Heft 1. Berlin und Leipzig, Vereinigung Wissenschalflicher Verleger, 1919, 12+368 pp.

The Heft opens with a fine portrait and a seven page appreciation of Emil Lampe's life and work. He was an editor of the "*Fortschritte*" since Jahrgang 1883. The Heft covers History and philosophy, Algebra, Arithmetic, and about twenty five pages of the fourth section on Combinatory analysis and the calculus of probabilities. The number of pages for the first three sections is about eighty more than for the corresponding sections of Jahrgang 1913, and about sixty more than for a similar portion of Jahrgang 1912.

The Theory of the Imaginary in Geometry together with the Trigonometry of the Imaginary. By J. L. S. HATTON, Cambridge, at the University Press, 1920. Royal 8vo. 8 + 216 pp. Price 18 shillings.

Preface: "The position of any real point in space may be determined by means of three real coördinates, and any three real quantities may be regarded as determining the position of such a point. In geometry as in other branches of pure mathematics the question naturally arises, whether the quantities concerned need necessarily be real. What, it may be asked, is the nature of the geometry in which the coördinates of any point may be complex quantities of the form $x + ix'$, $y + iy'$, $z + iz'$? Such a geometry contains as a particular case the Geometry of real points. From it the geometry of real points may be deduced (a) by regarding x', y', z' as zero, (b) by regarding x, y, z as zero, or (c) by considering only those points, the coördinates of which are real multiples of the same complex quantity $a + ib$. The relationship of the more generalized conception of geometry and of space to the particular case of real geometry is of importance, as points, whose determining elements are complex quantities, arise both in coördinate and in projective geometry.

"In this book an attempt has been made to work out and determine this relationship. Either of two methods might have been adopted. It would have been possible to lay down certain axioms and premises and to have developed a general theory therefrom. This has been done by other authors. The alternative method, which has been employed here, is to add to the axioms of real geometry certain additional assumptions. From these, by means of the methods and principles of real Geometry, an extension of the existing ideas and conceptions of geometry can be obtained. In this way the reader is able to approach the simpler and more concrete theorems in the first instance, and step by step the well-known theorems are extended and generalized. A conception of the imaginary is thus gradually built up and the relationship between the imaginary and the real is exemplified and developed. The theory as here set forth may be regarded from the analytical point of view as an exposition of the oft quoted but seldom explained 'Principle of Continuity.'

"The fundamental definition of Imaginary points is that given by Dr. Karl v. Staudt in his *Beiträge zur Geometrie der Lage;* Nuremberg, 1856 and 1860. The idea of (α, β) figures, independently evolved by the author, is due to J. V. Poncelet, who published it in his *Traité des Propriétés Projectives des Figures* in 1822. The matter contained in four or five pages of Chapter II is taken from the lectures delivered by the late Professor Esson, F.R.S., Savilian professor of geometry in the University of Oxford, and may be partly traced to the writings of v. Staudt. For the remainder of the book the author must take the responsibility. Inaccuracies and inconsistencies may have crept in, but long experience has taught him that these will be found to be due to his own deficiencies and not to fundamental defects in the theory. Those who approach